



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

cube roots of unity. Then the roots are $m+n$, $\omega m+\omega^2 n$, $\omega^2 m+\omega n$. When $b^2/4=a^3/27$, or $a^3/b^2=\frac{27}{4}=6.75$, $m=n=\pm\frac{1}{2}\sqrt[3]{(4b)}$.

$$\therefore m+n=\pm\sqrt[3]{(4b)}, \quad \omega m+\omega^2 n=\mp\frac{1}{2}\sqrt[3]{(4b)}, \quad \omega^2 m+\omega n=\mp\frac{1}{2}\sqrt[3]{(4b)}.$$

When $b^2/4 < a^3/27$, or $a^3/b^2 < \frac{27}{4}=6.75$, $\sqrt{(b^2/4-a^3/27)}$ is imaginary.

Let $m=u+\sqrt{(-1)v}$, $n=u-\sqrt{(-1)v}$.

$\therefore m+n=u+v$, $\omega m+\omega^2 n=-u-v\sqrt{3}$, $\omega^2 m+\omega n=-u+v\sqrt{3}$, all real and unequal.

When $a^3/b^2 > \frac{27}{4}=6.75$, $\sqrt{\frac{b^2}{4}-\frac{a^3}{27}}$ is real.

$\therefore m+n$ is real, and $\omega m+\omega^2 n$, $\omega^2 m+\omega n$ are imaginary.

J. W. Clawson, of Collegeville, Pa., referred to Burnside and Panton's *Theory of Equations*, Vol. I, §§42, 43. Discussions of this problem are to be found in nearly all texts on the Theory of Equations.

296. Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Sum the series, $1+\frac{1}{6}+\frac{1}{20}+\frac{1}{50}+\frac{1}{105}+\frac{1}{168}+\frac{1}{252}+\dots$

Solution by A. R. MAXSON, A. M., Columbia University, New York.

In the series 1, 6, 20, 50, 105, 196, 336, ..., the successive orders of differences are,

$$\begin{array}{ccccccc} 5, & 14, & 30, & 55, & 91, & 140, & \dots \\ 9, & 16, & 25, & 36, & 49, & & \dots \\ 7, & 9, & 11, & 13, & & & \dots \\ 2, & 2, & 2, & & & & \dots \\ 0, & 0, & & & & & \dots \end{array}$$

The n th term is then

$$\begin{aligned} 1+5(n-1)+\frac{9}{2!}(n-1)(n-2)+\frac{7}{3!}(n-1)(n-2)(n-3) \\ +\frac{2}{4!}(n-1)(n-2)(n-3)(n-4)=\frac{n}{12}(n+2)(n+1)^2. \end{aligned}$$

The n th term of the given series is then $\frac{12}{n(n+2)(n+1)^2}$, which can be written $\left(\frac{6}{n}+\frac{6}{n+1}\right)-\left(\frac{6}{n+1}-\frac{6}{n+2}\right)-\frac{12}{(n+1)^2}$.

Taking now u_r as the r th term of the original series, we have

$$u_1 = \left(\frac{6}{1} + \frac{6}{2}\right) - \left(\frac{6}{2} + \frac{6}{3}\right) - 12 \cdot \frac{1}{2^2},$$

$$u_2 = \left(\frac{6}{2} + \frac{6}{3}\right) - \left(\frac{6}{3} + \frac{6}{4}\right) - 12 \cdot \frac{1}{3^2},$$

$$\dots \dots \dots \dots \dots \dots \dots \dots$$

$$u_{n-1} = \left(\frac{6}{n-1} + \frac{6}{n}\right) - \left(\frac{6}{n} + \frac{6}{n+1}\right) - 12 \cdot \frac{1}{n^2}.$$

By addition, we have $\sum_{r=1}^{r=n-1} u_r = 9 - \frac{6}{n} - \frac{6}{n+1} + 12 - 12 \sum_{r=1}^{r=n-1} \frac{1}{r^2}$. For the sum to infinity we have $\sum_{r=1}^{r=\infty} u_r = 21 - 12 \cdot \frac{\pi^2}{6} = 21 - 2\pi^2$, on remembering that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

Also solved by G. B. M. Zerr, J. W. Clawson, and H. V. Spunar.

297. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.

If a, b, c, d, f, g, h are all real, and $a, ab-h^2, abc+2fgh-af^2-bg^2-ch^2$ are all positive, show that $b, c, bc-f^2$, and $ca-g^2$ are also positive.

I. Solution by C. R. MacINNIS, Princeton, N. J.

Since both a and $ab-h^2$ are positive, b must be positive.

$$abc+2fgh-af^2-bg^2-ch^2 \equiv \frac{(ab-h^2)(bc-f^2)-(hf-bg)^2}{b}.$$

Since the whole expression is positive and both b and $ab-h^2$ are also positive, $bc-f^2 > 0$. Hence $c > 0$. Similarly,

$$abc+2fgh-af^2-bg^2-ch^2 \equiv \frac{(ab-h^2)(ca-g^2)-(hg-af)^2}{a}, \text{ and } ca-g^2 > 0.$$

II. Solution by A. F. CARPENTER, Hastings, Nebr.

Since $ab-h^2$ is positive $ab > h^2$, and since h is real, h^2 is positive. Then ab , which is greater than h^2 , is positive. But a is positive; hence b is positive.

Now $b(abc+2fgh-af^2-bg^2-ch^2) = (ab-h^2)(bc-f^2) - (bg-fh)^2$; that is, $(bc-f^2)(ab-h^2) = b$ (a positive quantity) $+ (bg-fh)^2 = a$ positive quantity, and since $ab-h^2$ is positive, $(bc-f^2)$ is positive.

Again, $a(abc+2fgh-af^2-bg^2-ch^2) = (ab-h^2)(ca-g^2) - (af-hg)^2$, and it follows as before that $(ca-g^2)$ is positive.